

In this diagram we are building 1 octave scales. An octave is the distance between two pitches that are given the same letter name in traditional musical notation. Mathematically, a note an octave higher is roughly double the frequency of a note an octave lower. A note an octave lower is roughly half the frequency of a note an octave higher. Because the chromatic pitches are roughly equal distance from one another, simple algebra can be used to represent their notes and the relationship between one and the other.

In this system, X is the note you wish to start building a scale on. The Ys are all different notes of the scale. Scales vary in the amount of notes in them. We will primarily focus on the major and minor scales which are both seven note scales. Thus we will have seven different Y, (Y1-Y7), plus an additional Y1 at the end of the scale that is an octave above our starting note. Although the scale is represented from octave to octave (so 8 Ys), there are only 7 different notes (pitches, frequencies, degrees) in the scale, which is why it is called a seven note scale instead of an eight note scale.

To find our Y values, we take X plus a specific number (Z) that is different for each note of the scale to find all the notes of the scale. Our Z value represents the interval distance between the note we start on (our X value, called the tonic of the scale) and the note/degree of the scale we are trying to find.

The subscript number assigned to the different Ys (Y1, Y2, etc.) is a different numbering system based on the scale itself and how many notes are in it. "Degree" is another name for notes of the scale. So in a seven note scale for example, there are seven notes (called degrees). Scale degree 2 is the second note of the scale (Y2), scale degree 3 is the 3rd note of the scale (Y3), etc. In traditional music theory, the different degrees of the scale are represented by roman numerals, which I have also included in addition to the Y values.

This is a big part of where our I IV V nomenclature comes from. For example, if we build a major chord on the fourth degree (IV) of a major scale, this is the IV chord of the scale. Chord progressions are ultimately a result of the relationships between notes within each scale, but that is beyond the scope of this course. Capital roman numerals represent major chords (IV), lowercase roman numerals represent minor chords (iv).

X=Tonic of Scale

Z=interval distance between tonic and the degree of scale you are trying to find

Y=X+Z=Scale Degree

As a general rule, if your scale or chord starts on a sharp note, the other notes should be sharp notes or natural notes. If the scale or chord starts on a flat note, the other notes should be flat notes or natural notes.

Major Scale: 7 note scale.

If X=0, C, Do

Unfixed Solfege	Do	Re	Mi	Fa	So	La	Ti	Do	
Chord Numerals	I	ii	iii	IV	V	vi	vii ^o	I	
Scale Intervals	X+0	X+2	X+4	X+5	X+7	X+9	X+11	X+0	
Example Key X=0	Y1=0	Y2=2	Y3=4	Y4=5	Y5=7	Y6=9	Y7=11	Y1=0	
Note Letters	C	C	D	E	F	G	A	B	C

Minor Scale: another 7 note scale

If X=9, A, La

	Do	Re	Me	Fa	So	La	Ti	Do	
Chord Numerals	i	ii ^o	III	iv	v(V)	VI	VII	i	
Scale Intervals	X+0	X+2	X+3	X+5	X+7	X+8	X+10	X+0	
Example Key X=9	Y1=0	Y2=11	Y3=0	Y4=2	Y5=4	Y6=5	Y7=7	Y1=0	
Note Letters	A	A	B	C	D	E	F	G	C

As a point of reference, let's do the whole tone scale. The whole tone scale is a six note scale (Y1-Y6) that divides the 12 chromatic notes of the octave in half, creating two six note scales.

Whole Tone Scale, 6 Note Scale. There are only two whole tone scales.

Whole tone scale does not have numerals.

Scale Intervals	X+0	X+2	X+4	X+6	X+8	X+10	X+0
Example Key X=0	Y1=0	Y2=2	Y3=4	Y4=6	Y5=8	Y6=10	Y7=0
Note Letters	C	C	D	E	F#/Gb	G#/Ab	A#/Bb C

Scale Degree	Y1	Y2	Y3	Y4	Y5	Y6	Y1
Scale Intervals	X+0	X+2	X+4	X+6	X+8	X+10	X+0
Example Key X=1	Y1=1	Y2=3	Y3=5	Y4=7	Y5=9	Y6=11	Y1=1
Note Letters	C#/Db	C#/Db	D#/Eb	F	G	A	B C#/Db

The Pentatonic Scales are 5 note scales (Y1-Y5)

Major Pentatonic

Unfixed Solfege	Do	Re	Mi	So	La	Do
Numerals:	I	ii	iii	V	vi	I
Scale Intervals	X+0	X+2	X+4	X+7	X+9	X+12
Example Key X=0	Y1=0	Y2=2	Y3=4	Y4=7	Y5=9	Y6=0
Note Letters	C	C	D	E	G	A

Minor Pentatonic Scale

Unfixed Solfege	Do	Me	Fa	So	Te	Do
Chord Numerals	i	III	iv	v(V)	VII	i
Scale Degree	Y1	Y2	Y3	Y4	Y5	Y1
Scale Intervals	X+0	X+3	X+5	X+7	X+10	X+0
Example Key X=9	Y1=9	Y2=0	Y3=2	Y4=4	Y5=7	Y6=9
Note Letters	A	A	C	D	E	G

Please note that every major scale has a corresponding minor scale that has the exact same notes. Also notice that the pentatonic scales are just major and minor scales with two notes removed. I included numerals in the pentatonic scales so you could more easily see what scale degrees are kept and what are removed.

The major pentatonic scale removes scale degrees VI (Y4) and viio (Y7) from the major scale. The minor pentatonic scale removes scale degrees ii0 (Y2) and VI (Y6) from the major scale.